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$\varepsilon'_K/\varepsilon_K$ in the chiral limit*

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Abstract: The $K \to \pi\pi$ system is analyzed in the chiral limit within the Standard Model. We discuss how to connect the short-distance running in the $|\Delta S| = 1$ case to the matrix-elements calculated in a low-energy approximation in a scheme-independent fashion. We calculate this correction and the resulting Wilson Coefficients. The matrix elements are calculated to next-to-leading order in the $1/N_c$ expansion and combined with the Wilson coefficients to calculate the two isospin amplitudes and ε'_K. The $\Delta I = 1/2$ rule is reproduced within expected errors and we obtain a substantially larger value for $\varepsilon'_K/\varepsilon_K$ than most other analysises. We discuss the reasons for this difference. We also suggest that the X-boson method is an option for lattice QCD calculations.

Keywords: 1/N Expansion, Weak Decays, Kaon Physics, CP-violation.

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Contents

1. Introduction

The study of CP-violation took a major step forward in 1999 with the first direct evidence by two independent experiments for direct CP-violation [1] corroborating the earlier NA31 result. The average of the two experiments

$$\left|\frac{\varepsilon'_K}{\varepsilon_K}\right|^{\text{exp}} = (1.92 \pm 0.46) \cdot 10^{-3}\,. \tag{1.1}$$

is higher than the standard calculations obtained [2, 3] and this has caused claims for physics beyond the Standard Model, see e.g. [4]. The purpose of this paper is to calculate this quantity within our earlier proposed method and see whether the above number can be reached or not. Other opinions critical of this claim are expressed in [5].

CP-violation was first observed in the sixties and was then shown to proceed only via K^0-$\overline{K^0}$ mixing. This is called indirect CP-violation and is described by the parameter ε_K

$$\varepsilon_K = \frac{A\left[K_L \to (\pi\pi)_{I=0}\right]}{A\left[K_S \to (\pi\pi)_{I=0}\right]}\,. \tag{1.2}$$

In addition there is also direct CP-violation in the decay-amplitude possible in $K \to \pi\pi$ decays. This is parameterized by

$$\varepsilon'_K = \frac{1}{\sqrt{2}}\left[\frac{A\left[K_L \to (\pi\pi)_{I=2}\right]}{A\left[K_S \to (\pi\pi)_{I=0}\right]} - \frac{A\left[K_L \to (\pi\pi)_{I=0}\right]}{A\left[K_S \to (\pi\pi)_{I=0}\right]}\frac{A\left[K_S \to (\pi\pi)_{I=2}\right]}{A\left[K_S \to (\pi\pi)_{I=0}\right]}\right]. \qquad (1.3)$$

Combining both definitions the ratio is directly accessible from

$$\frac{\varepsilon'_K}{\varepsilon_K} = \frac{1}{\sqrt{2}}\left[\frac{A\left[K_L \to (\pi\pi)_{I=2}\right]}{A\left[K_L \to (\pi\pi)_{I=0}\right]} - \frac{A\left[K_S \to (\pi\pi)_{I=2}\right]}{A\left[K_S \to (\pi\pi)_{I=0}\right]}\right]. \qquad (1.4)$$

This last formula is the basis for the double-ratio method used in the actual experiments [1].

Theoretical calculations of $\varepsilon'_K/\varepsilon_K$ basically reduce to calculating the $K \to \pi\pi$ amplitudes. This is in the Standard Model a several step process, see section 3 and e.g. the lectures of [6] for more details. First the heavy particles, top, Z and W, are integrated out, resulting in an effective action in terms of four-quark operators. Then the renormalization group evolution equations are used to bring the coefficients in this effective action down to a hadronic scale thus resumming large logarithms containing heavy masses. This part has been performed including gluonic and electroweak Penguin-diagrams to one-loop in [7] and to two-loops in [8].

This has then been used with matrix-elements from the lattice and from the first $1/N_c$-methods [9] to calculate $\varepsilon'_K/\varepsilon_K$ in [2]. We will not comment further on the lattice calculations. A recent suggestion about their calculation and more references can be found in [10]. We will concentrate on the $1/N_c$ method of calculating the matrix-elements.

The $1/N_c$-method was first proposed in [9] after it was noticed that using large N_c instead of vacuum-insertion-approximation for the matrix elements gave a significant improvement for the $\Delta I = 1/2$ rule. They simply identified the cut-off in meson-loops with the scale in the renormalization group. A better way to provide this identification was given in [11, 12] by using colour-singlet bosons to provide the identification of scales. To one-loop it was proven in [12] that this procedure gives the correct matching. The work of [13] is essentially the continuation of [9] using this identification directly with the output of the renormalization group.

Our work has concentrated on two more improvements. We realized that our method of using colour-singlet bosons, hereafter referred to as the X-boson method, allowed both a solution to the scale-identification and to the scheme-dependence that appears at two-loops in the renormalization group. This was argued in [14] where we used available results to include part of the needed correction. We proved this explicitly in the case of B_K in [15] and below for the $|\Delta S = 1|$ case. The other improvement we have included is the use of the ENJL model for the couplings of the X-bosons to improve on the high-energy behaviour. For B_K this can be done directly and we presented results including all quark-mass corrections in [12, 15].

For $K \to \pi\pi$ decays this produced too unwieldy calculations and we use instead the method of [16] to reduce the calculation to off-shell two-point functions. Results for the operators Q_1 to Q_6 in the chiral limit were presented in [14] and in the same limit for Q_7 to Q_{10} below.

Related work, concentrating on a more QCD inspired improvement of the X-couplings is ref. [17]. We regard this method as very promising but it has not yet reached the stage where results for the quantities considered here are possible.

An approach where no attempt at precisely identifying the scales in the matrix-element and the short-distance running is done is the chiral quark model approach of [18]. On the other hand this series of papers has attempted to include several effects neglected by the standard analysises [2] by systematically going to p^4 and thus provided an indication that the errors in [2] were severely underestimated.

Another approach to determining the non-leptonic matrix-elements is to use QCD sum rules to calculate more inclusive quantities and then relate those to the matrix-elements. Examples of this approach are [19].

As discussed in [12] for the case of the B_K parameter, a general problem with this method is the existence of operators in the CHPT description that contribute to the inclusive quantities but not to the decays $K \to \pi\pi$. The reason is that the so-called spurion-fields have now non-vanishing derivatives.[1]

We would like to stress that our method basically reproduces the $\Delta I = 1/2$ rule, has no free input parameters as several other models have and includes a correct scheme and scale identification at all stages. As will become obvious, we also reproduce the value of $\varepsilon'_K/\varepsilon_K$ within the uncertainty of our approach.

Our X-boson method can be used in general and is not restricted to $1/N_c$. e.g. by adapting the method of [20] to use X-bosons instead of photons it might provide an alternative lattice QCD method to calculate $K \to \pi\pi$ on the lattice.

In the next section we introduce the basic notation used to describe $K \to \pi\pi$ decays to lowest order in Chiral Perturbation Theory (CHPT). Section 3 describes the X-boson method and calculates the corrections needed to identify the scales in QCD and the low-energy model used. The main result is presented in the appendix. Section 4 presents the calculation in $1/N_c$ using CHPT and the ENJL model for the X-boson couplings for the electroweak Penguin operators. The numerical input used is summarized in section 5. The results from the previous sections and the matrix-elements of [14] are then combined to calculate the parameters in the CHPT $\Delta S = 1$ lagrangian in section 6 and presented using the more standard bag parameters, B_i in section 7. The next section contains the calculation of ε'_K and briefly of ε_K as well. The last section discusses the result for $\varepsilon'_K/\varepsilon_K$ and adds the two corrections which are expected to dominate.

[1] In the notation of section 2 an example of an extra octet term is $\mathrm{tr}(\Delta_{32}u_\mu)\,\mathrm{tr}(\nabla^\mu(\Delta_{11}+\Delta_{22}+\Delta_{33}))$.

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2. Chiral perturbation theory lagrangian

We use the standard Chiral Perturbation Theory (CHPT) counting with p a momentum, energy or meson-mass and e the electromagnetic coupling constant $e=\sqrt{4\pi\alpha}$. The lowest-order strong and electromagnetic lagrangian is given by

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\,\mathrm{tr}\left(u_\mu\, u^\mu + \chi_+\right) + e^2\,\tilde{C}_2\,\mathrm{tr}\left(QUQU^\dagger\right) \tag{2.1}$$

with $U=u^2=\exp(\lambda^a\pi^a/F_0)$ and $u_\mu = iu^\dagger D_\mu U u^\dagger$. λ^a are the Gell-Mann matrices and the π^a are the pseudoscalar-mesons π, K and η. $Q=\mathrm{diag}(2/3,-1/3,-1/3)$ is the light-quark-charge matrix and $\chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u$. $\chi = 2B_0\,\mathrm{diag}(m_u,m_d,m_s)$ collects the light-quark masses. To this order $F_0=F_\pi$ is the pion decay coupling constant. An introduction to CHPT can be found in the lectures [21].

To order e^0p^2 and e^2p^0, the chiral lagrangian describing $|\Delta S|=1$ transitions is given by

$$\begin{aligned}\mathcal{L}^{(2)}_{|\Delta S|=1} = C\,F_0^6\,e^2\,G_E\,\mathrm{tr}\left(\Delta_{32}\tilde{Q}\right) + CF_0^4\Big[&G_8\,\mathrm{tr}\left(\Delta_{32}u_\mu\,u^\mu\right) + G_8'\,\mathrm{tr}\left(\Delta_{32}\chi_+\right) + \\ &+ G_{27}\,t^{ij,kl}\,\mathrm{tr}\left(\Delta_{ij}U_\mu\right)\mathrm{tr}\left(\Delta_{kl}u^\mu\right)\Big] + \text{h.c.}\end{aligned} \tag{2.2}$$

with $\Delta_{ij}=u\lambda_{ij}u^\dagger$, $(\lambda_{ij})_{ab}=\delta_{ia}\,\delta_{jb}$, $\tilde{Q}=u^\dagger Q u$;

$$C = -\frac{3}{5}\,\frac{G_F}{\sqrt{2}}\,V_{ud}V_{us}^* \approx -1.06\cdot 10^{-6}\,\mathrm{GeV}^{-2}\,. \tag{2.3}$$

The SU(3) $\times$ SU(3) tensor $t^{ij,kl}$ can be found in [16].

The $K\to\pi\pi$ invariant amplitudes can be decomposed into definite isospin quantum numbers amplitudes as $[A\equiv -iT]$

$$\begin{aligned}A\left[K_S\to\pi^0\pi^0\right] &\equiv \sqrt{\frac{2}{3}}\,A_0 - \frac{2}{\sqrt{3}}\,A_2\,,\\ A\left[K_S\to\pi^+\pi^-\right] &\equiv \sqrt{\frac{2}{3}}\,A_0 + \frac{1}{\sqrt{3}}\,A_2\,,\\ A\left[K^+\to\pi^+\pi^0\right] &\equiv \frac{\sqrt{3}}{2}\,A_2\,,\end{aligned} \tag{2.4}$$

where $K_S\simeq K_1^0+\varepsilon_K K_2^0$; $K^0_{1(2)}\equiv (K^0-(+)\overline{K}^0)/\sqrt{2}$ and $\mathrm{CP}(K^0_{1(2)}) = +(-)K^0_{1(2)}$. The final state interaction phases δ_0 and δ_2 are included into the amplitudes A_0 and A_2 as follows:

$$\begin{aligned}A_0 &\equiv -i\,a_0\,e^{i\delta_0} \quad \text{for the isospin 1/2 amplitude};\\ A_2 &\equiv -i\,a_2\,e^{i\delta_2} \quad \text{for the isospin 3/2 amplitude}.\end{aligned} \tag{2.5}$$

Using the lagrangian of eq. (2.2) we obtain

$$
\begin{aligned}
a_0 &= \frac{\sqrt{6}}{9}\, C F_0 \Big[(9\, G_8 + G_{27}) \left(m_K^2 - m_\pi^2\right) - 6\, e^2\, G_E\, F_0^2 \Big] \,, \\
a_2 &= \frac{\sqrt{3}}{9}\, C F_0 \Big[10\, G_{27} \left(m_K^2 - m_\pi^2\right) - 6\, e^2\, G_E\, F_0^2 \Big] \,.
\end{aligned} \tag{2.6}
$$

In the presence of CP-violation the couplings G_8, G_{27} and G_E get an imaginary part. In the Standard Model, $\mathrm{Im}\, G_{27}$ vanishes and $\mathrm{Im}\, G_8$ and $\mathrm{Im}\, G_E$ are proportional to $\mathrm{Im}\, \tau$ with $\tau \equiv -\lambda_t/\lambda_u$ and $\lambda_i \equiv V_{id} V_{is}^*$.

3. Short-distance

In section 4 we will calculate the long-distance contributions in the X-boson scheme. This scheme requires only the matching of currents between the low-energy model/theory and the short-distance QCD calculations. This way the scheme- and scale-dependence inherent in most of the other calculations is consistently avoided. This scheme has been shown to one-loop to reproduce the correct matching [12] and was argued to work to all orders in [14]. The precise proof to two-loops was done in [15] for the case of the B_K-parameter. Exactly the same calculation can be performed for the $|\Delta S = 1|$ case. There is no new problem but in practice the calculation is much longer because there are now ten operators instead of one. Penguin diagrams need to be taken into account and photonic loops as well as gluonic ones have to be considered. We will therefore present fewer explicit expressions than in [15].

We start from the effective action that is the output of the renormalization group running at two-loops which is given by

$$
\Gamma_{\mathrm{eff}} = -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^* \int d^4x \sum_{i=1,10} C_i\, Q_i(x) \,, \tag{3.1}
$$

with $C_i = z_i + y_i \tau$ and

$$
\begin{aligned}
Q_1 &= (\bar{s}_\alpha \gamma_\mu u_\beta)_L (\bar{u}_\beta \gamma^\mu d_\alpha)_L \,, & Q_2 &= (\bar{s}_\alpha \gamma_\mu u_\alpha)_L (\bar{u}_\beta \gamma^\mu d_\beta)_L \,, \\
Q_3 &= (\bar{s}_\alpha \gamma_\mu d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta \gamma^\mu q_\beta)_L \,, & Q_4 &= (\bar{s}_\alpha \gamma_\mu d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta \gamma^\mu q_\alpha)_L \,, \\
Q_5 &= (\bar{s}_\alpha \gamma_\mu d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta \gamma^\mu q_\beta)_R \,, & Q_6 &= (\bar{s}_\alpha \gamma_\mu d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta \gamma^\mu q_\alpha)_R \,, \\
Q_7 &= (\bar{s}_\alpha \gamma_\mu d_\alpha)_L \sum_{q=u,d,s} \frac{3}{2} e_q (\bar{q}_\beta \gamma^\mu q_\beta)_R \,, & Q_8 &= (\bar{s}_\alpha \gamma_\mu d_\beta)_L \sum_{q=u,d,s} \frac{3}{2} e_q (\bar{q}_\beta \gamma^\mu q_\alpha)_R \,, \\
Q_9 &= (\bar{s}_\alpha \gamma_\mu d_\alpha)_L \sum_{q=u,d,s} \frac{3}{2} e_q (\bar{q}_\beta \gamma^\mu q_\beta)_L \,, & Q_{10} &= (\bar{s}_\alpha \gamma_\mu d_\beta)_L \sum_{q=u,d,s} \frac{3}{2} e_q (\bar{q}_\beta \gamma^\mu q_\alpha)_L
\end{aligned} \tag{3.2}
$$

with $(\bar{q}\gamma_\mu q')_{(L,R)} = \bar{q}\gamma_\mu(1 \mp \gamma_5) q'$; α and β are colour indices.

The Wilson coefficients $C_i(\mu_R)$ are dependent on the scheme chosen for γ_5, the choice of evanescent operators, the scale μ_R in the renormalization group as well as other scheme choices like the choice of infrared regulators. All of them have to be consistently treated when calculating matrix elements in order to reach a physical result. Reviews of this can be found in [3] and [6]. The main original references are [8]. We have chosen two slightly different implementations of the running, both in the NDR scheme. One where we linearize fully the NLO part, this way all divergences appearing in the expressions can be analytically resolved[2] and the scheme dependence is canceled exactly except for effects of order $\alpha_S(m_c)^2$. The other implementation is where we exactly solve the two-loop evolution equations for the $C_i(\mu_R)$. In this case the scheme-dependence is only canceled to order $\alpha_S(\mu_R)^2$. We regard the difference between these two as a rough estimate of the uncertainty due to higher order corrections in $\alpha_S(\mu_R)$.

The next step is now to replace the effective action (3.1) by another equivalent effective action in $D = 4$ where the quarks only appear in vector, axial-vector currents and scalar or pseudoscalar densities. These we know how to hadronize. Specifically we replace eq. (3.1) by

$$
\begin{aligned}
\Gamma_X = {} & g_1 X_1^\mu \Big((\bar s\,\gamma_\mu\, d)_L + (\bar u\,\gamma_\mu\, u)_L \Big) + g_2\, X_2^\mu \Big((\bar s\,\gamma_\mu\, u)_L + (\bar u\,\gamma_\mu\, d)_L \Big) + \\
& + g_3\, X_3^\mu \left((\bar s\,\gamma_\mu\, d)_L + \sum_{q=u,d,s} (\bar q\,\gamma_\mu\, q)_L \right) + g_4 \sum_{q=u,d,s} X_{q,4}^\mu \Big((\bar s\,\gamma_\mu\, q)_L + (\bar q \gamma_\mu q)_L \Big) + \\
& + g_5\, X_5^\mu \left((\bar s\,\gamma_\mu\, d)_L + \sum_{q=u,d,s} (\bar q\,\gamma_\mu\, q)_R \right) + g_6 \sum_{q=u,d,s} X_{q,6} \Big((\bar s\, q)_L + (-2)(\bar q\, d)_R \Big) + \\
& + g_7\, X_7^\mu \left((\bar s\,\gamma_\mu\, d)_L + \sum_{q=u,d,s} \frac{3}{2}\, e_q\, (\bar q\,\gamma_\mu\, q)_R \right) + \\
& + g_8 \sum_{q=u,d,s} X_{q,8} \left((\bar s\, q)_L + (-2) \frac{3}{2}\, e_q\, (\bar q\, d)_R \right) + \\
& + g_9\, X_9^\mu \left((\bar s\,\gamma_\mu\, d)_L + \sum_{q=u,d,s} \frac{3}{2}\, e_q\, (\bar q\,\gamma_\mu\, q)_L \right) + \\
& + g_{10} \sum_{q=u,d,s} X_{q,10}^\mu \left((\bar s\,\gamma_\mu\, q)_L + \frac{3}{2}\, e_q\, (\bar q\,\gamma_\mu\, q)_L \right) .
\end{aligned}
\tag{3.3}
$$

Here all colour sums are performed implicitly inside the brackets and $(\bar q q')_{(L,R)} = \bar q (1 \mp \gamma_5) q'$.

For simplicity we choose all X-bosons to have the same mass. We now determine the couplings g_i as a function of the C_i by taking matrix elements of both sides

[2] Note that in none of the papers of [8] all necessary formulae were given, we have worked them out ourselves.

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between quark and gluon external states, labelled by ψ and ψ' and require

$$\langle\psi'|\, e^{i\Gamma_{\rm eff}}\, |\psi\rangle = \langle\psi'|\, e^{i\Gamma_{\rm X}}\, |\psi\rangle + \mathcal{O}\left(\frac{1}{M_X^4}\right). \tag{3.4}$$

Both the left- and right-hand side can be written in terms of the tree level matrix-elements of the Q_i between the states ψ, ψ'. Leading to

$$\begin{aligned}
&\sum_j \Bigg[\delta_{ij} + \frac{\alpha_s(\mu_R)}{\pi}\left(\gamma_{ij}\log\frac{m}{\mu_R} + r_{ij} + F_{ij}\left(m^2,\psi,\psi'\right)\right) + \\
&\qquad + \frac{\alpha}{\pi}\left(\gamma^e_{ij}\log\frac{m}{\mu_R} + r^e_{ij} + F^e_{ij}\left(m^2,\psi,\psi'\right)\right)\Bigg] C_j(\mu_R) = \\
&= \sum_j \Bigg[\delta_{ij} + \frac{\alpha_s(\mu_C)}{\pi}\left((\gamma_{ij} - \tilde{\gamma}_{ij})\log\frac{m}{M_X} + \tilde{\gamma}_{ij}\log\frac{m}{\mu_C} + \tilde{r}_{ij} + F_{ij}\left(m^2,\psi,\psi'\right)\right) + \\
&\qquad + \frac{\alpha}{\pi}\left(\left(\gamma^e_{ij} - \tilde{\gamma}^e_{ij}\right)\log\frac{m}{M_X} + \tilde{\gamma}^e_{ij}\log\frac{m}{\mu_C} + \tilde{r}^e_{ij} + F^e_{ij}\left(m^2,\psi,\psi'\right)\right)\Bigg] \frac{g_j(\mu_C)^2}{M_X^2}.
\end{aligned} \tag{3.5}$$

The parameter m is some IR regulator. For conserved currents $\tilde{\gamma}_{ij} = 0$ and there are no logarithms of the scale μ_C. For non-conserved currents or densities $\tilde{\gamma}_{ij}$ can be related directly to their anomalous dimensions.

The infrared sensitive parts are in the functions F_{ij} and in F^e_{ij} and are identical on both sides. We have checked this explicitly for a set of states similar to the one chosen in [15]. The resulting formulas are very long and only reinforce the argument for complete infrared ambiguity cancellation. Notice that the matrices γ_{ij} and γ^e_{ij} are one-loop anomalous dimension matrices of the set of four-quark operators (3.2) [7]. The left-hand side is thus completely scheme-independent to order α_S^2. The scheme-dependence of the $C_j(\mu_R)$ is canceled by the scheme-dependence of the constants r_{ij} and r^e_{ij}. The right-hand side is similarly scale-independent

From eq. (3.5) we can now obtain

$$\begin{aligned}
\frac{g_i(\mu_C)^2}{M_X^2} = \sum_j \Bigg[&\delta_{ij} + \frac{\alpha_s(\mu_C)}{\pi}\left(\gamma_{ij}\log\frac{M_X}{\mu_R} + \tilde{\gamma}_{ij}\log\frac{\mu_C}{M_X} + r_{ij} - \tilde{r}_{ij}\right) + \\
&+ \frac{\alpha}{\pi}\left(\gamma^e_{ij}\log\frac{M_X}{\mu_R} + \tilde{\gamma}^e_{ij}\log\frac{\mu_C}{M_X} + r^e_{ij} - \tilde{r}^e_{ij}\right) + \\
&+ O\Big(\alpha_S\left(\mu_R\right) - \alpha_S\left(\mu_C\right)\Big)\Bigg] C_j(\mu_R)\,.
\end{aligned} \tag{3.6}$$

The X-boson couplings are now completely scheme- and scale-independent and the large logarithms of M_W are resummed and included in the values of the couplings g_i. The explicit expressions for the differences $r_{ij} - \tilde{r}_{ij}$ and $r^e_{ij} - \tilde{r}^e_{ij}$ are given in appendix A.

We have not used any large N_c arguments up to this point. Now we need to calculate the matrix-elements of X-boson exchange between hadronic external states.

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As described in more detail in [15] and references therein, we do this by rotating the integral over the X-boson momenta into euclidean space and splitting it into two-parts separated by an euclidean cut-off μ.

The large momentum part of this integral [between μ and ∞] can be simply calculated. The large X-boson momentum must flow back via quark- or gluon-lines and can be calculated perturbatively. The part where the momentum flow-back is through the hadronic wave-functions is suppressed by $1/\mu^2$ and can be neglected. Precisely as described in [15] the result is proportional to $\alpha_S(\mu)$ and is thus suppressed by $1/N_c$ compared to the tree-level matrix elements or it is down by an extra factor of α and reads

$$\begin{aligned}\langle\text{out}|X_j\text{-exchange}|\text{in}\rangle^{\text{Short-distance}} &= \sum_i \left[\frac{\alpha_S(\mu)}{\pi}\left(\gamma_{ij}-\tilde{\gamma}_{ij}\right)+\frac{\alpha}{\pi}\left(\gamma^e_{ij}-\tilde{\gamma}^e_{ij}\right)\right]\times \\ &\quad\times\log\frac{M_X}{\mu}\,\langle\text{out}|X_i\text{-exchange}|\text{in}\rangle_{\text{Leading }1/N_c}\,.\end{aligned} \tag{3.7}$$

This part of the integration can thus be included by replacing in eq. (3.6) everywhere $\log M_X$ by $\log\mu$ leading to

$$\begin{aligned}\left(\frac{g_i(\mu_C,\mu)^2}{M_X^2}\right)_{\text{eff}} &= \sum_j\left[\delta_{ij}+\frac{\alpha_s(\mu_C)}{\pi}\left(\gamma_{ij}\log\frac{\mu}{\mu_R}+\tilde{\gamma}_{ij}\log\frac{\mu_C}{\mu}+r_{ij}-\tilde{r}_{ij}\right)+\right.\\ &\quad+\frac{\alpha}{\pi}\left(\gamma^e_{ij}\log\frac{\mu}{\mu_R}+\tilde{\gamma}^e_{ij}\log\frac{\mu_C}{\mu}+r^e_{ij}-\tilde{r}^e_{ij}\right)+\\ &\quad\left.+O\Big(\alpha_S\left(\mu_R\right)-\alpha_S\left(\mu_C\right)\Big)\right]C_j(\mu_R)\,.\end{aligned} \tag{3.8}$$

It is obvious from this that the final answer for the decays will not depend on the artificial choice of the mass M_X.

The low-energy part of the integral [between 0 and μ] over the X-boson momenta must now be performed in a particular low-energy approximation. We must therefore identify correctly the vector, axial-vector currents and scalar and pseudoscalar densities appearing in eq. (3.3) in the low-energy model. This is no problem for the vector and axial-vector currents since they are currents from a conserved symmetry. For the scalar and pseudoscalar densities the situation is somewhat more uncertain. There is remaining dependence on $\log\mu_C$ from the terms with $\tilde{\gamma}_{ij}$ and $\tilde{\gamma}^e_{ij}$. The scale μ_C is the one at which currents and densities in QCD and in the low-energy model (CHPT and/or ENJL) are matched. We stress again that this problem of matching of the (pseudo)scalar densities to the low-energy model, though uncertain, is a much more tractable problem than the original matching of the four-quark operators of eq. (3.1) directly. Practically, we do this matching when substituting the CHPT or ENJL quark-condensate by the QCD value at $\mu_C\simeq 1\,\text{GeV}$. For other scales we run the quark density with its QCD anomalous dimension.

4. Long-distance

As described before in [12, 14] we can calculate the very low-energy part using CHPT to lowest order for the X-boson couplings.[3] For the intermediate energy region one can use hadronic models for the X-boson couplings. They must have the correct chiral symmetry and reproduce at very low energy the model-independent CHPT results. Here we will use the ENJL model. As we noted earlier in [14, 15] this provides a substantial improvement over using lowest-order CHPT X-boson couplings. It brings in *no new* undetermined parameters, describes a fairly wide range of low-energy hadronic phenomena [22] and satisfies some short-distance constraints from QCD as well [23].

The contributions to order e^0p^2 were already worked out in [14]. We now add the results relevant to order e^2p^0. They can be calculated using our earlier methods, a well-defined off-shell K-π Green function [16], or with a reduction method as was used in [17]. The comparison between both and a more extensive discussion of the results in this section will be presented elsewhere [24].

Via the strong interaction, the coupling $G_E F_0^6$ gets contributions only from Q_7 and Q_8. This result follows simply from the fact that gluonic exchanges cannot change the chiral properties of an operator and only Q_7 and Q_8 are $8_L \times 8_R$ under chiral symmetry. However, if long-distance photonic (or other electroweak) effects are included the other operators can contribute. This is extremely important for the real part of this coupling where Q_1 and Q_2 contribute and have much larger coefficients than the others. In the imaginary part of $e^2 G_E$, the main other operator is Q_6 which has a coefficient such that long-distance photonic effects in its matrix-elements have a smaller effect on $\mathrm{Im}\, G_E$. Our approximation corresponds to including photonic-electroweak effects at a scale from μ to ∞ and neglecting the photonic contribution only from 0 to μ. We expect this as explained above to be a reasonable approximation for the imaginary part of $e^2 G_E$ and not for the real part. Notice that $\mathrm{Re}\, e^2 G_E$ contributes only to CP conserving transitions and is a small effect.

The Wilson coefficient C_7 is order 1 in $1/N_c$ and C_8 is order $1/N_c$. The leading in $1/N_c$ contribution to the matrix element of Q_8 comes from the two diagrams in figure 1 and is of order N_c^2. Using quark-loop diagrams one can prove model-independently [24] that in the chiral limit the next-to-leading order in $1/N_c$ vanishes to all orders in X-boson couplings. The non-factorizable contributions from Q_8 thus only start at order e^2p^2 and do not contribute to G_E. This contribution leads to $G_E F_0^6$ of order N_c.

[3] Note that we use lowest-order CHPT in two different meanings. Lowest order in CHPT in the couplings in the lagrangian of eq. (2.2) and lowest order in CHPT in the X-boson couplings which are used to calculate the coefficients in eq. (2.2).

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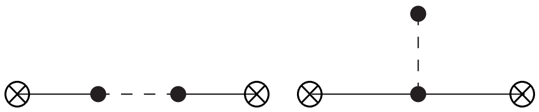

Figure 1: The lowest order, e^2p^0, diagrams for the contributions from Q_8. $\otimes$ is an insertion of the pseudo-scalar currents. The dashed line is an X-boson and the dots are the X-boson-pseudoscalar couplings. Full lines are pseudo-scalar propagators.

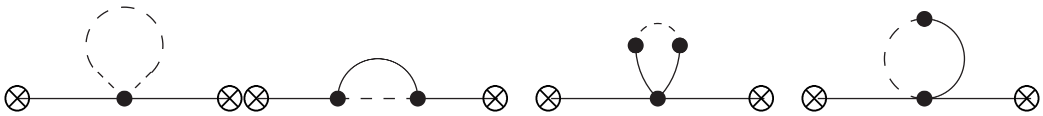

Figure 2: The lowest order, e^2p^0, diagrams for the contributions from Q_7 but next-to-leading in $1/N_c$. Notation is as in figure 1.

The leading contribution in $1/N_c$ from Q_7 is factorizable but is only of order e^2p^2. The next-to-leading contribution in $1/N_c$ contributes to order e^2p^0 at order N_c to the matrix-element and leads again to $G_E F_0^6$ of order N_c. The N_c counting presented here is scheme-independent.

We can now calculate the leading contributions using lowest-order CHPT for the X-boson couplings and obtain

$$e^2\, G_E\,[Q_8] = -5\,\frac{B_0^2(\mu)}{F_0^2}\,C_8(\mu) = -5\,\frac{\langle 0|\,\overline{q}\,q\,|0\rangle^2(\mu)}{F_0^6}\,C_8(\mu)\,, \tag{4.1}$$

where the Wilson coefficient is the one appropriate for our X-boson scheme as discussed in section 3 and $\langle 0|\overline{q}q|0\rangle(\mu) \equiv -F_0^2\,B_0(\mu)$ is the quark condensate in the chiral limit. There are also $O(e^2)$ contributions from Q_8 to G_8 and G_{27}.

The calculations from the diagrams in figure 2 lead to

$$e^2\,G_E\,[Q_7] = -\frac{15}{2}\,\frac{\mu^4}{16\,\pi^2 F_0^4}\,C_7(\mu)\,. \tag{4.2}$$

Both results in eqs. (4.1) and (4.2) agree with those of [17] when the results for $e^2G_E[Q_7]$ listed there are restricted to lowest-order CHPT only. Notice that the formula (20) in that reference is only valid for $\Lambda^2 \gg M_A^2, M_V^2$ and can only be applied at much higher scales. That approximation partially explains the large scale dependence found below 1.3 GeV in [17]. Λ there corresponds to μ here.

We stress again that the Wilson coefficients in (4.1) and (4.2) are in the X-boson scheme where the matrix-elements can be identified unambiguously.

We can now turn to the ENJL model for the X-boson couplings. These results have a better high-energy behaviour. As mentioned already above, for Q_8 the next-to-leading order in $1/N_c$ vanishes and the ENJL result thus coincides with eq. (4.1). The result for $e^2G_E[Q_7](\mu)/C_7(\mu)$ is given in table 1. The numbers should be compared with the lowest order result for $e^2G_E[Q_8]^{\mathrm{ENJL}} = -5040\,C_8(\mu)$ using $B_0^{\mathrm{ENJL}} = 2.8\,\mathrm{GeV}$.

μ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
$\frac{e^2\, G_E[Q_7](\mu)}{C_7(\mu)}$	-0.1	-1.5	-6.9	-19.6	-42.5	-77.5	-126	-189	-266	-358

Table 1: The ENJL model results for the contribution from Q_7 to $e^2 G_E(\mu)$ in the X-boson scheme.

The latter needs to be corrected for the QCD value of $B_0(1\,\mathrm{GeV}) = 1.75\,\mathrm{GeV}$ [25] and QCD running. This correction is the one discussed in section 3 in order to identify currents and densities in QCD and in the low-energy model and is unambiguous. The usual assumption on the value of m_s needed shows up in our way in the value of B_0, i.e. the quark condensate in the chiral limit. The relevant quantity is really the value of the quark condensate[4] rather than the value of m_s.

5. Numerical input

The input values we use are

$$\begin{aligned}
&\overline{m}_t(m_t) = (165 \pm 5)\,\mathrm{GeV}\,; \qquad \overline{m}_b(m_b) = 4.4\,\mathrm{GeV}\,; \qquad \overline{m}_c(m_c) = 1.23\,\mathrm{GeV}\,;\\
&\sin^2(\theta_W) = 0.2315\,\mathrm{GeV}\,; \qquad M_W = 80.3945\,\mathrm{GeV}\,; \qquad M_Z = 91.1872\,\mathrm{GeV}\,;\\
&|V_{us}| = 0.2196\,; \qquad |V_{cb}| = 0.040 \pm 0.002\,; \qquad \left|\frac{V_{ub}}{V_{cb}}\right| = 0.090 \pm 0.025\,.
\end{aligned} \tag{5.1}$$

Quark masses are in the $\overline{\mathrm{MS}}$ scheme, M_Z and M_W are pole masses. The electroweak input values have been taken from the review by Sirlin [27], the CKM matrix values from the review by Falk [28].

We will simply take $\sin(\delta) = 1$ so that

$$\mathrm{Im}\,\tau = -6.72 \cdot 10^{-4}\,. \tag{5.2}$$

In addition we use two sets of values for the strong coupling constant. Set I we choose with the central value of the measurement at the τ-mass, $\alpha_S(m_\tau) = 0.345$ [29] and set II with the world average value at M_Z, $\alpha_S(M_Z) = 0.1186$ [27]. We use exact running with the two-loop beta function, this is what is most closely needed for the running of the Wilson coefficients.

The scale-independent coefficients $\hat{\eta}_{i=1,3}$ needed for the calculation of ε_K we take from [30], for $\hat{\eta}_1$ we interpolate the values given in table 5 of that reference and obtain $\hat{\eta}_1 = 1.93$ (Set I) and $\hat{\eta}_1 = 1.53$ (set II). The others are $\hat{\eta}_2 = 0.57$ and $\hat{\eta}_3 = 0.47$.

[4] In fact, the next-to-leading order CHPT corrections proportional to quark masses are not the ones necessary to change $\langle 0|\overline{q}q|0\rangle$ into $\langle 0|\overline{s}s|0\rangle$ [14, 26]. So the matrix element of Q_6 and Q_8 at higher orders are not proportional to the strange quark condensate squared or to the inverse squared strange quark mass via PCAC. Interpreting the value of B_0 in terms of m_s via lowest-order PCAC as done in the usual $\varepsilon'_K/\varepsilon_K$ analyses it corresponds to $m_s(1\,\mathrm{GeV}) \approx 140\,\mathrm{MeV}$.

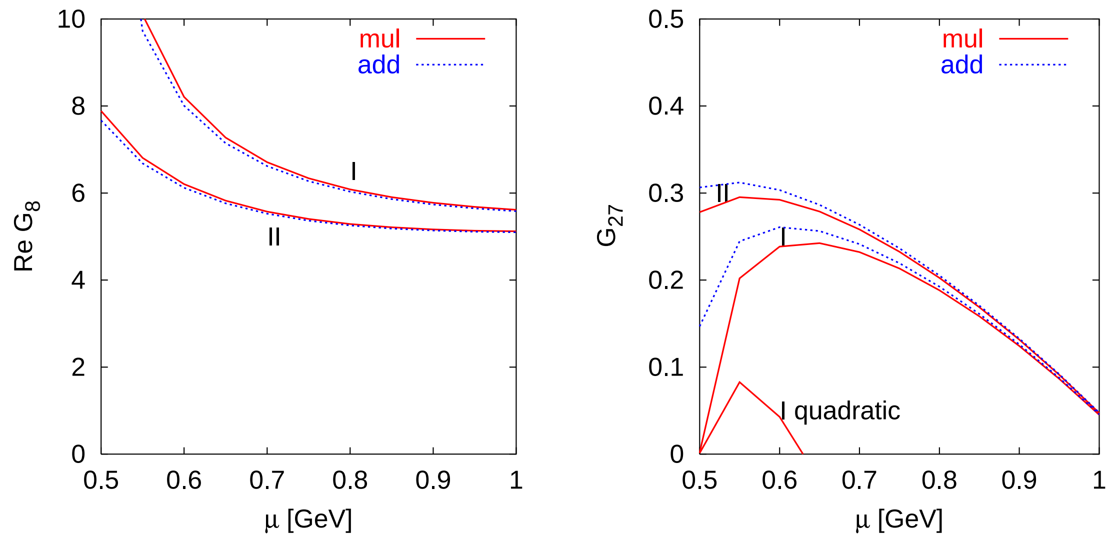

Figure 3: The real part of G_8 and G_{27} as a function of μ. The experimental value of $\mathrm{Re}\, G_8$ is about 6.2 and for G_{27} about 0.48. I and II refer to the two values of α_S used and the two curves to two ways of solving the evolution equations as described in the text. In the large-N_c limit both G_{27} and $\mathrm{Re}\, G_8$ are equal to 1.

In addition we use $F_K = 112.7\,\mathrm{MeV}$, $\hat{B}_K = 0.77 \pm 0.10$ [15], $F_0 = 87\,\mathrm{MeV}$, $B_0^{\mathrm{QCD}}(1\,\mathrm{GeV}) = 1.75\,\mathrm{GeV}$ and the parameters of the ENJL model as determined by the fit in [22].

6. Results for G_8, G_{27} and G_E

We now combine the matrix-elements and the Wilson coefficients in the X-boson scheme to obtain the quantities needed for the $\Delta I = 1/2$ rule and $\varepsilon'_K/\varepsilon_K$.

We use here the input values given above and the two sets of values for α_S, labelled I and II. In order to have some estimate of higher order corrections in α_S we have treated the NLO running of the Wilson coefficients and the transition to the X-boson scheme in two ways which only differ in higher orders in α_S. One where we use the exact solution of the two-loop evolution equations and multiply with the corrections to go from the NDR to the X-boson scheme. This is labelled with *mul* for multiplicative in the figures. The other option is to only approximately solve the NLO running as a series in α_S. We also include the scheme-correction now additively with the running. This is labelled *add* in the figures. Other estimates give a similar variation.

For definiteness we also set all the three scales μ_R, μ_C and μ equal to each other. The variation with this assumption for B_K was shown in [15] and is smaller than the difference between the two values of α_S used.

The results relevant for the $\Delta I = 1/2$ rule are shown in figure 3. For $\mathrm{Re}\, G_8$ we obtain a reasonable matching and a value which is in reasonable agreement with the experimental value of 6.2. It is dominated by the contributions from Q_1 and Q_2. A large part can be considered as the low-energy Penguins as we have discussed in [14]. For G_{27} we do not have good stability. The ENJL model improves very strongly on just using CHPT for the X-boson couplings as used in the work of the Dortmund group, but it still provides a too strong suppression. The curve labelled *I quadratic* in figure 3 shows the CHPT approximation for the X-boson couplings. The ENJL improvement is obvious.

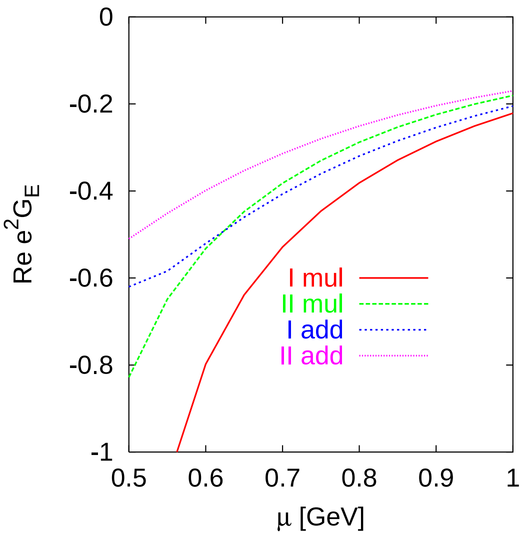

Figure 4: The real part $\mathrm{Re}\, e^2 G_E$ as a function of μ. I and II refer to the two values of α_S used and the two curves for each value of α_S are for the two ways of solving the evolution equations as described in the text. No stability is expected as explained in the text.

As explained above we do not expect stability for the real part of G_E since here long-distance photon corrections to the Q_1 and Q_2 are expected to be important and our approximations only include photonic contributions from short-distances. Nevertheless we show our result for $\mathrm{Re}(e^2 G_E)$ in figure 4 for completeness.

The parts multiplying τ of G_8 and $e^2 G_E$ are shown in figure 5. Notice that the strong contribution proceeding mainly through Q_6 and dominating $\mathrm{Im}\, G_8$ is from the gluonic Penguin. The electroweak Penguins contribute via $e^2 G_E$ and the contribution is dominated by Q_8. The stability of both results is quite acceptable. These numbers are the input used for ε'_K below.

7. B-parameters in the NDR scheme

The matrix-elements are usually quoted in terms of the bag-parameters. These are defined as the ratio of the matrix-elements divided by the vacuum-insertion-approximation (VIA) matrix-elements. The latter have no scheme-dependence and so the bag-parameters are also scheme-dependent. From our earlier results the bag-parameters can be most easily determined in the X-boson scheme. Afterwards they need to be converted to the more usual NDR-scheme.

The usual definitions as given in e.g. [6, 17] are

$$B_1^{(1/2)}(\mu) \equiv \frac{M_0\,[Q_1]}{\mathcal{A}}\,; \qquad B_2^{(1/2)}(\mu) \equiv -\frac{1}{5}\,\frac{M_0\,[Q_2]}{\mathcal{A}}\,;$$

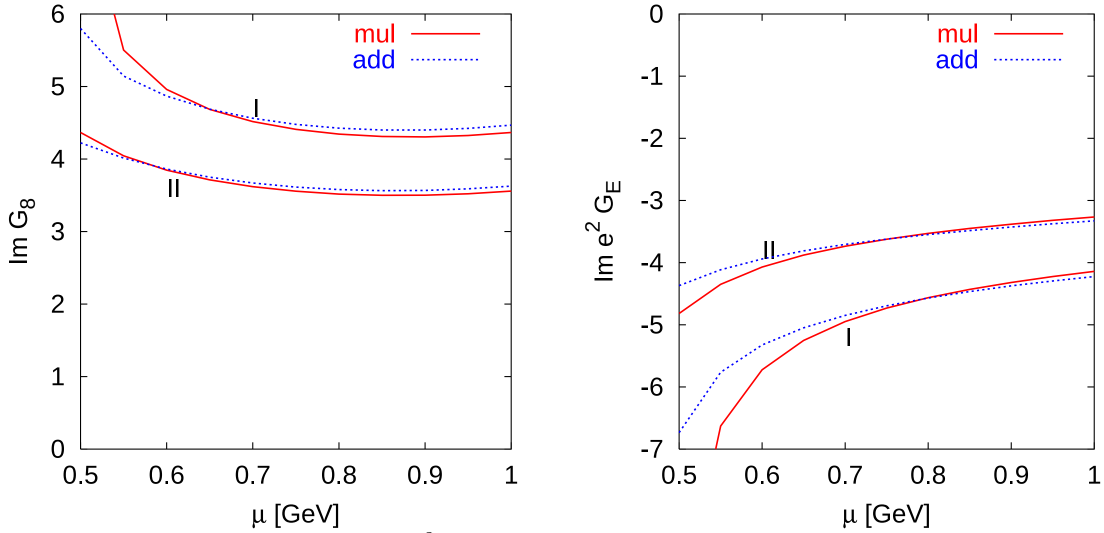

Figure 5: The part of G_8 and e^2G_E multiplying τ as a function of μ. I and II refer to the two values of α_S used and the two curves for each value of α_S are for the two ways of solving the evolution equations as described in the text.

$$
\begin{aligned}
&B_1^{(3/2)}(\mu) = B_2^{(3/2)}(\mu) = -\frac{\sqrt{2}}{8}\,\frac{M_2[Q_1]}{\mathcal{A}}\,; \qquad B_6^{(1/2)}(\mu) = \frac{M_0\,[Q_6]}{\mathcal{C}}\,;\\
&B_7^{(1/2)}(\mu) = 3\,\frac{M_0\,[Q_7]}{\mathcal{D}}\,; \qquad B_7^{(3/2)}(\mu) = 3\,\sqrt{2}\,\frac{M_2\,[Q_7]}{\mathcal{D}}\,;\\
&B_8^{(1/2)}(\mu) = \frac{M_0\,[Q_8]}{\mathcal{D}}\,; \qquad B_8^{(3/2)}(\mu) = \sqrt{2}\,\frac{M_2\,[Q_8]}{\mathcal{D}}\,;
\end{aligned}
\tag{7.1}
$$

with

$$
M_0\,[Q_j] \equiv \langle(\pi\pi)_{I=0}|Q_j|K^0\rangle\,, \qquad M_2\,[Q_j] \equiv \langle(\pi\pi)_{I=2}|Q_j|K^0\rangle
\tag{7.2}
$$

and

$$
\begin{aligned}
&\mathcal{A} \equiv -\frac{\sqrt{3}}{9}\,F_0\left(m_K^2 - m_\pi^2\right), \qquad \mathcal{C} \equiv -16\,\sqrt{3}\,L_5\,(M_\rho)\,\frac{\langle 0|\,\overline{q}\,q\,|0\rangle^2(\mu)}{F_0^5}\left(m_K^2 - m_\pi^2\right),\\
&\mathcal{D} \equiv -2\,\sqrt{3}\,\frac{\langle 0|\,\overline{q}\,q\,|0\rangle^2(\mu)}{F_0^3}\,.
\end{aligned}
\tag{7.3}
$$

We have restricted the definition of [6] here to the lowest order in CHPT for consistency. Full expressions away from the chiral limit are in the first reference of [2].

In the large-N_c limit one gets the model independent results [9, 31]

$$
\begin{aligned}
&B_1^{(1/2)} = 3\,; \qquad B_2^{(1/2)} = \frac{6}{5}\,; \qquad B_1^{(3/2)} = B_2^{(3/2)} = \frac{3}{4}\,;\\
&B_6^{(1/2)} = B_8^{(3/2)} = B_8^{(1/2)} = 1\,;
\end{aligned}
\tag{7.4}
$$

and

$$
B_7^{(3/2)} = B_7^{(1/2)} = 0\,.
\tag{7.5}
$$

Again these results are valid in the chiral limit. The operators Q_7 and Q_8 are discussed more below.

We normalized the B_7-parameter to the Fierzed part of the vacuum insertion approximation. This definition of the B_7-parameter is the same as used by the lattice community [32], [17] and in [33]. Notice that in some cases terms beyond lowest order are included in their definitions.

To lowest order in CHPT, i.e. $\mathcal{O}(e^0p^2)$ or $\mathcal{O}(e^2p^2)$ for the weak lagrangian one can rewrite the definitions of eq. (7.1) in the notation used here and in [14] for the matrix-elements:

$$
\begin{aligned}
B_{1\chi}^{(1/2)}(\mu) &= -\frac{3}{5\,C_1(\mu)}\left(9\,G_8\,[Q_1]\,(\mu) + G_{27}\,[Q_1]\,(\mu)\right);\\
B_{2\chi}^{(1/2)}(\mu) &= \frac{3}{25\,C_2(\mu)}\left(9\,G_8\,[Q_2]\,(\mu) + G_{27}\,[Q_2]\,(\mu)\right);\\
B_{1\chi}^{(3/2)}(\mu) &= B_{2\chi}^{(3/2)}(\mu) = \frac{3}{4\,C_1(\mu)}\,G_{27}\,[Q_1]\,(\mu)\,;\\
B_{6\chi}^{(1/2)}(\mu) &= -\frac{3\,F_0^6}{80\,\langle 0|\,\overline{q}\,q\,|0\rangle^2(\mu)\,L_5(M_\rho)}\;\frac{G_8\,[Q_6]\,(\mu)}{C_6(\mu)}\,;\\
B_{7\chi}^{(3/2)}(\mu) &= B_{7\chi}^{(1/2)}(\mu) = -\frac{3\,F_0^6}{5\,\langle 0|\,\overline{q}\,q\,|0\rangle^2(\mu)}\;\frac{e^2\,G_E\,[Q_7]\,(\mu)}{C_7(\mu)}\,;\\
B_{8\chi}^{(3/2)}(\mu) &= B_{8\chi}^{(1/2)}(\mu) = -\frac{F_0^6}{5\,\langle 0|\,\overline{q}\,q\,|0\rangle^2(\mu)}\;\frac{e^2\,G_E\,[Q_8]\,(\mu)}{C_8(\mu)}\,.
\end{aligned}
\tag{7.6}
$$

The others can be derived similarly but we have only given the numerically important ones and $B_7^{(3/2)}$ in order to compare with other work [17, 32, 33, 34]. We have also restricted the VIA matrix-elements to order e^0p^2 and e^2p^0 in order to be consistent with our full calculation.

Notice that even in the X-boson scheme the values of $B_6^{(1/2)}$, $B_7^{(I)}$ and $B_8^{(I)}$ depend on the value of α_S because of the running of $B_0(\mu)$. The leading order in $1/N_c$ running of the Wilson coefficients $C_6(\mu)$ and $C_8(\mu)$ is exactly cancelled by the scale dependence of the matrix elements for Q_6 and Q_8 calculated at the same order [31]. The bag-parameters $B_6(\mu)$ and $B_8(\mu)$ run in the same way with $B_0(\mu)$.

The large N_c result for the Q_7 operator is $O(e^2p^2)$ in CHPT and thus vanishes in the chiral limit. The lowest e^2p^0 order and numerically larger contribution is NLO in $1/N_c$ and survives in the chiral limit. This operator together with Q_8 have been studied by several groups and techniques and deserves special attention and will studied within the present approach in [24].

The numerical results for the bag-parameters in the X-boson scheme can be easily derived using the results from section 4 and ref. [14], in the case of lowest-order CHPT couplings of the X-bosons using eqs. (4.1) and (4.2) and ref. [14, eqs. (5.2) and (5.17)]. In the case of the ENJL model couplings for the X-bosons one needs to use eq. (4.1), table 1 and ref. [14, tables 1, 2 and 3]. Notice that $B_{8\chi}^{(I)X} = 1$ in the X-boson scheme even at NLO in $1/N_c$ while $B_{6\chi}^{(1/2)X} = 2.2 \pm 0.5$ [26].

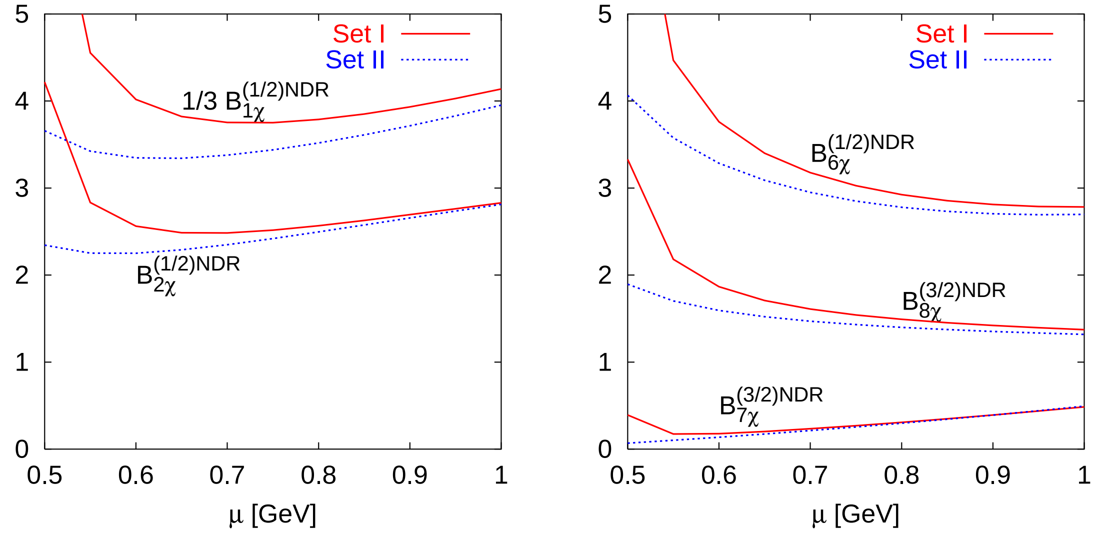

Figure 6: The bag parameters in the NDR scheme for the two values of α_S we use. $B_{1\chi}^{(1/2)\mathrm{NDR}}$ has been divided by 3 to make it fit better in the plot. Notice the large values for $B_{1\chi}^{(1/2)\mathrm{NDR}}$ and $B_{2\chi}^{(1/2)\mathrm{NDR}}$ responsible for the $\Delta I = 1/2$ rule and the large value for $B_{6\chi}^{(1/2)\mathrm{NDR}}$ which is the reason for our large value of $\varepsilon'_K/\varepsilon_K$.

In order to get to the NDR-scheme we multiply with the combination $r_{ij} - \tilde{r}_{ij}$ discussed in section 3 via

$$B_i^{(I)\mathrm{NDR}} = \frac{1}{\langle Q_i\rangle^I_{\mathrm{VIA}}} \sum_j \left[\delta_{ij} - \frac{\alpha_S(\mu)}{\pi}\left(r_{ij} - \tilde{r}_{ij}\right) - \frac{\alpha}{\pi}\left(r^e_{ij} - \tilde{r}^e_{ij}\right)\right] B_j^{(I)X} \langle Q_j\rangle^I_{\mathrm{VIA}} \,. \quad (7.7)$$

The $B_j^{(I)X}$-parameters are in the X-boson scheme. We show the resulting numerical results for the inputs given earlier in figure 6. Notice the large values for $B_{1\chi}^{(1/2)\mathrm{NDR}}$ and $B_{2\chi}^{(1/2)\mathrm{NDR}}$ responsible for the $\Delta I = 1/2$ rule and the large value for $B_{6\chi}^{(1/2)\mathrm{NDR}}$ which is the reason for our large value of $\varepsilon'_K/\varepsilon_K$ while $B_{8\chi}^{(3/2)\mathrm{NDR}}$ shows no such large enhancement. Notice though that we get $B_{8\chi}^{(3/2)\mathrm{NDR}}(\mu) \approx 1.4 \pm 0.2$ for μ between 0.6 GeV and 1 GeV which is around 30% to 40% larger than most other analysises.

In the NDR scheme and in the chiral limit we get $B_{7\chi}^{(3/2)\mathrm{NDR}}(\mu) \approx 0.3 \pm 0.2$ for scales μ between 0.6 GeV and 1 GeV.

In order to compare with previous work from other groups we now use the renormalization group to run these B-factors to 2 GeV and obtain the results shown in figure 7. Here we can see that the result for G_8 which contains a sum of contributions with B_1 and B_2 will probably be quite stable but G_{27} which contains a difference will be very unstable since it is a difference between large numbers. This is of course precisely what we observed above.

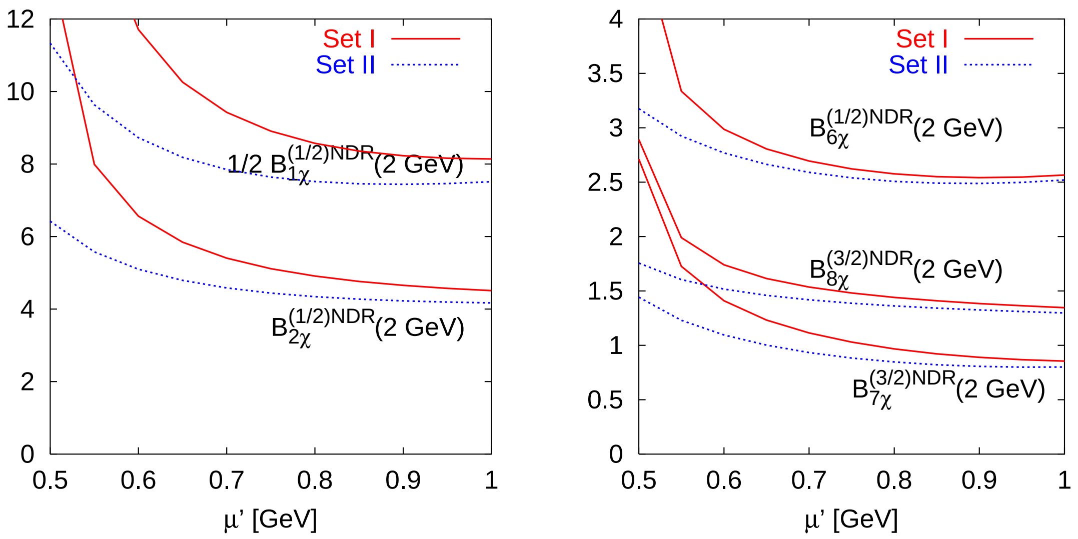

Figure 7: The bag parameters in the NDR scheme for the two values of α_S we use at a scale of $\mu = 2\,\mathrm{GeV}$. $B_{1\chi}^{(1/2)\mathrm{NDR}}$ has been divided by 2 to make it fit better in the plot. We have used our matrix-elements at μ' and then run to $\mu = 2\,\mathrm{GeV}$.

Method	$B_6^{(1/2)\mathrm{NDR}}(2\,\mathrm{GeV})$	$B_7^{(3/2)\mathrm{NDR}}(2\,\mathrm{GeV})$	$B_8^{(3/2)\mathrm{NDR}}(2\,\mathrm{GeV})$
This work [χ limit]	2.5 ± 0.4	0.8 ± 0.1	1.35 ± 0.20
LMD [17]	–	0.94	–
Dispersive [33]	–	0.78 ± 0.17	1.6 ± 0.4
QCD sum rules [34]	2.99 ± 0.34	0.7 ± 0.2	1.70 ± 0.39
CHPT large N_c [13]	$1.5 \sim 1.7$	$-0.1 \sim 0.09$	$0.4 \sim 0.7$
Lattice [32]	–	$0.5 \sim 0.8$	$0.7 \sim 1.1$
Chiral QM [18]	$1.2 \sim 1.7$	~ 0.9	~ 0.9

Table 2: Results for $B_6^{(1/2)\mathrm{NDR}}$, $B_7^{(3/2)\mathrm{NDR}}$ and $B_8^{(3/2)\mathrm{NDR}}$ at a scale of $2\,\mathrm{GeV}$. The value $m_s(2\,\mathrm{GeV}) = (119 \pm 12)\,\mathrm{MeV}$ was used in [34] and for rescaling the results in [17, 33].

We can now compare the other bag-parameters with previous work. Notice that in some cases the numbers also include chiral corrections. We have used the values quoted in ref. [34] to have them all at the same basis, since all of them are rather sensitive to the value of the quark condensate used, or alternatively to the value of m_s. Notice that all the analytical approaches that have control of the μ-dependence and have models/data that describe hadronic interactions in the intermediate scales agree well with each other for $B_7^{(3/2)\mathrm{NDR}}$ and $B_8^{(3/2)\mathrm{NDR}}$. The two recent calculations of $B_6^{(1/2)\mathrm{NDR}}$ are also in good agreement.

8. Results for ε_K and ε'_K to lowest order

The indirect CP-violation as described by ε_K, defined by eq. (1.2), can be rewritten to first order in CP-violating parameters, using the $\Delta I = 1/2$ rule and with $\mathrm{Re}\, a_0 \gg \varepsilon_K\ \mathrm{Im}\, a_0$, to a very good approximation as

$$\begin{aligned}\varepsilon_K &\simeq \frac{e^{i\pi/4}}{\sqrt{2}}\left(\frac{\mathrm{Im}\, M_{12}}{\Delta m_K} + \frac{\mathrm{Im}\, a_0}{\mathrm{Re}\, a_0}\right),\\ M_{12} &= \frac{G_F^2}{6\,\pi^2} F_K^2\, \hat{B}_K\, m_K\, M_W^2 \Big[(\lambda_c^*)^2\, \hat{\eta}_1\, S(x_c) + (\lambda_t^*)^2\, \hat{\eta}_2\, S(x_t) + \\ &\qquad + 2\,\lambda_c^*\,\lambda_t^*\,\hat{\eta}_3\, S\left(x_c, x_t\right)\Big]\,. \end{aligned} \tag{8.1}$$

Putting[5] in the central numerical values of section 5, $\sin(\delta) = 1$, $\mathrm{Re}\, G_8 = 6.2$ and $G_{27} = 0.48$ we obtain with $e^2 G_E$ and $\mathrm{Im}\, G_8$ calculated in section 6

$$\begin{aligned}|\varepsilon_K| &= (2.40 - 0.34)\cdot 10^{-3} = 2.06\cdot 10^{-3} \quad \text{Set I;}\\ |\varepsilon_K| &= (2.49 - 0.28)\cdot 10^{-3} = 2.21\cdot 10^{-3} \quad \text{Set II}\end{aligned} \tag{8.2}$$

at a value of $\mu = 0.75\,\mathrm{GeV}$. The μ-dependence is very mild. The first (second) number in brackets is the first (second) term in eq. (8.1). The main difference is in the value of $\hat{\eta}_1$. The effect from the second term which is more dependent on the quantities calculated here, is small.

Given the uncertainty on $\hat{B}_K$ and the other input parameters this result agrees well with the experimental value

$$\varepsilon_K^{\exp} = e^{i\Phi_{\varepsilon_K}}\,(2.280 \pm 0.013)\cdot 10^{-3} \tag{8.3}$$

with $\Phi_{\varepsilon_K} = (43.49 \pm 0.08)^0$. For definiteness we will below use the value of eq. (8.3) for our numerical estimates of $\varepsilon'_K/\varepsilon_K$.

Using the same inputs we can now calculate ε'_K from its definition eq. (1.3). To first order in CP-violating parameters, using the $\Delta I = 1/2$ rule and with $\mathrm{Re}\, a_0 \gg \varepsilon_K\ \mathrm{Im}\, a_0$, we can use to a very good approximation

$$\varepsilon'_K \simeq \frac{1}{\sqrt{2}}\,\frac{\mathrm{Re}\, a_2}{\mathrm{Re}\, a_0}\left(-\frac{\mathrm{Im}\, a_0}{\mathrm{Re}\, a_0} + \frac{\mathrm{Im}\, a_2}{\mathrm{Re}\, a_2}\right)\, e^{i\Phi_{\varepsilon'_K}}\,. \tag{8.4}$$

With $\Phi_{\varepsilon'_K} = \pi/2 + \delta_2 - \delta_0 = (45.8 \pm 3.1)^o$ [35] the strong phase.

$$\begin{aligned}\left|\frac{\varepsilon'_K}{\varepsilon_K}\right| &= (9.2 - 2.5)\cdot 10^{-3} = 6.7\cdot 10^{-3} \quad \text{Set I}\,,\\ \left|\frac{\varepsilon'_K}{\varepsilon_K}\right| &= (7.4 - 1.9)\cdot 10^{-3} = 5.5\cdot 10^{-3} \quad \text{Set II}\end{aligned} \tag{8.5}$$

[5] We use the experimental value of G_{27} to avoid the large μ-dependence seen in our calculation of that quantity. All other quantities were more stable as seen in section 6. In our earlier work on electromagnetic effects and as shown here for G_8, whenever we had stability we agreed within errors with experiment.

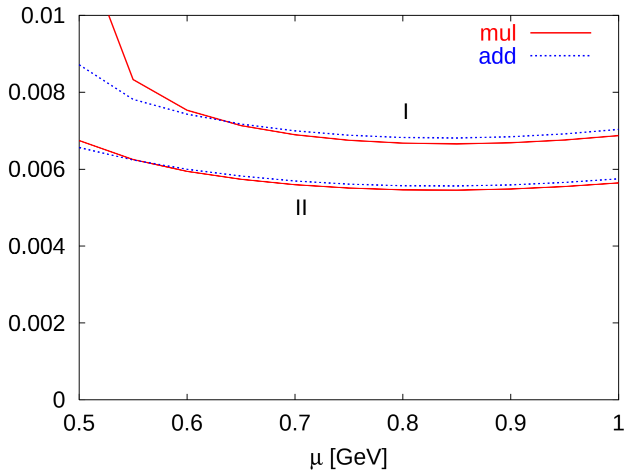

Figure 8: Our result for $\varepsilon'_K/\varepsilon_K$ in the chiral limit using the experimental values of ε_K and the inputs as described in the text. I and II refer to the two values of α_S used and the two curves to two ways of solving the evolution equations as described in the text.

at a value of $\mu = 0.75\,\mathrm{GeV}$. The first (second) number in brackets is the first (second) term in eq. (8.4). The first number is dominated by Q_6 and is usually called the strong Penguin contribution. The second one is from the contribution of Q_7 to Q_{10} and is dominated by Q_8 and is usually called the electroweak Penguin contribution.

We have also shown the μ-dependence of the final result for the two values of α_S and the two ways of NLO resumming used in figure 8. The predictions are quite stable, partly due to the fact that we fully calculate all contributions in the same way.

Our final prediction to lowest order in CHPT is

$$\left|\frac{\varepsilon'_K}{\varepsilon_K}\right|_{O(p^2)} = (6 \pm 3) \cdot 10^{-3}\,. \tag{8.6}$$

We used the experimental value of ε_K in the above numbers but using the values we calculated would not change any results dramatically.

The error we quoted there rest on several things, the matrix-elements of Q_6 and Q_8 depend directly on $B_0^2(1\,\mathrm{GeV})$, this does not affect the relative size of Q_6 and Q_8 but does affect the overall value in addition to the typical 30% or so we expect our method to have; we did not include any uncertainties in CKM matrix elements and as discussed below, there are several more effects which we did not include yet.

9. Discussion and corrections

Our number is significantly higher than any others discussed in the recent literature [4]. The strong Penguin in our case is enhanced in two ways compared to the usual treatment. First the scheme where the transitions to models can be made must be taken into account. This provides a first enhancement. Then in the X-boson scheme and in the chiral limit the B_6-parameter takes a value of $B_{6\chi}^X \approx 2.2 \pm 0.5$ [26] and $B_{8\chi}^X = 1$ at next-to-leading order in $1/N_c$. Putting it together with the first effect leads to $B_{6\chi}^{\mathrm{NDR}}(\mu) \approx 2.8 \pm 0.6$ in the chiral limit for scales between 0.6 GeV and 1 GeV as discussed above. We do not see any similar enhancement for $B_{8\chi}^{\mathrm{NDR}}(\mu) \approx 1.4 \pm 0.2$, so the usual strong cancellation between the strong and electroweak Penguins is much weaker in our calculation. A last reason is that we treat both the real and imaginary part at the same order, not as is usually done by treating the real part to all orders using experiment and the imaginary part to lowest order.

To the value above one has to add higher order CHPT corrections which are proportional to pion and Kaon masses. These are of three types, namely:

(1) final state interactions (FSI) from $\pi - \pi$ interactions

(2) CHPT corrections which are purely real, they are due to $\Delta S = 1$ higher order terms in the lagrangian and real loop-diagrams;

(3) isospin breaking corrections due to quark masses, usually estimated by $\pi_0 - \eta$ mixing, and electromagnetic corrections.

FSI have recently been considered in ref. [36] and later also in [37]. The main point of [36] is that both the real and the imaginary part of the amplitudes should be treated in the same way leading to a significant enhancement over the usual estimates. We already treat the imaginary and the real parts in eq. (8.4) in the same way so the main effect of FSI is in changing the ratio $\mathrm{Re}\, a_0 / \mathrm{Re}\, a_2$ from about 16.2 to 22.2. Including only FSI to *all orders* then changes eq. (8.5) to

$$\left| \frac{\varepsilon_K'}{\varepsilon_K} \right| = (6.8 - 1.8) \cdot 10^{-3} = 5.0 \cdot 10^{-3} \quad \text{Set I}\,,$$
$$\left| \frac{\varepsilon_K'}{\varepsilon_K} \right| = (5.4 - 1.4) \cdot 10^{-3} = 4.0 \cdot 10^{-3} \quad \text{Set II} \tag{9.1}$$

Isospin breaking effects due to the $\pi^0 - \eta$ mixing to $O(p^4)$ have been calculated in ref. [38] and references therein. This effect adds a contribution to $\mathrm{Im}\, a_2$ which is parameterized usually as

$$\frac{[\mathrm{Im}\, a_2]_{IB}}{\mathrm{Re}\, a_2} = \Omega_{IB} \frac{\mathrm{Im}\, a_0}{\mathrm{Re}\, a_0}\,. \tag{9.2}$$

Using the value $\Omega_{IB} = 0.16 \pm 0.03$ [38] one gets

$$\left| \frac{\varepsilon_K'}{\varepsilon_K} \right| = (6.8 - 2.9) \cdot 10^{-3} = 3.9 \cdot 10^{-3} \quad \text{Set I}\,,$$
$$\left| \frac{\varepsilon_K'}{\varepsilon_K} \right| = (5.4 - 2.3) \cdot 10^{-3} = 3.1 \cdot 10^{-3} \quad \text{Set II} \tag{9.3}$$

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Corrections of type (2) have been fitted to $O(p^4)$ in [39] and calculated to the same order in [18] but disentangling this from their FSI effects is not obvious. Notice that in [18] the enhancement found for B_6 is from this source and not from the one discussed here.

Isospin breaking corrections beyond mixing have been discussed recently as well, see [40], but their full estimate is not done at the present stage. Our approach allows to include them in a more systematic fashion and this will be done in the future.

Given the uncertainties on the CKM matrix-elements which we did not include and the uncertainties on the hadronic matrix elements which we still face, a reasonable guess at the final result is

$$\left|\frac{\varepsilon'_K}{\varepsilon_K}\right| = (3.4 \pm 1.8)\cdot 10^{-3}\,. \tag{9.4}$$

This result is somewhat above the present world average [1]

$$\left|\frac{\varepsilon'_K}{\varepsilon_K}\right|^{\rm exp} = (1.92 \pm 0.46)\cdot 10^{-3} \tag{9.5}$$

but quite compatible with it.

In conclusion we have calculated the chiral limit prediction for $\varepsilon'_K/\varepsilon_K$ in a consistent fashion using the $1/N_c$ expansion and the X-boson method. The result is substantially larger than most other estimates. After including the two largest expected corrections we obtain a result in acceptable agreement with the experiment.

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A. Scheme dependence

The matching conditions of eq. (3.5) between the effective lagrangian action (3.1) and the effective X-boson (3.3) in section 3 include two ten by ten matrices. The explicit calculation gives for the gluonic corrections

$$r - \tilde{r} = \frac{1}{4}\times \tag{A.1}$$

$$
\times \begin{pmatrix}
\frac{11}{2N_c} & \frac{-11}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{-11}{2} & \frac{11}{2N_c} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-7}{18N_c} & \frac{85}{18N_c} & -\frac{99N_c+n}{18N_c} & 0 & \frac{4n}{9N_c} & 0 & \frac{4\tilde{n}}{9N_c} & \frac{7}{18N_c} & -\frac{\tilde{n}}{18N_c} \\
0 & \frac{7}{18} & \frac{85}{18} & \frac{99+nN_c}{18N_c} & 0 & -\frac{4n}{9} & 0 & -\frac{4\tilde{n}}{9} & -\frac{7}{18} & \frac{\tilde{n}}{18} \\
0 & -\frac{7}{18N_c} & -\frac{7}{9N_c} & -\frac{n}{18N_c} & \frac{1}{2N_c} & \frac{-27N_c+4n}{9N_c} & 0 & \frac{4\tilde{n}}{9N_c} & \frac{7}{18N_c} & -\frac{\tilde{n}}{18N_c} \\
0 & \frac{7}{18} & \frac{7}{9} & \frac{n}{18} & -\frac{1}{2} & \frac{27-4nN_c}{9N_c}+5C_F & 0 & -\frac{4\tilde{n}}{9} & -\frac{7}{18} & \frac{\tilde{n}}{18} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2N_c} & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & \frac{3}{N_c}+5C_F & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{2N_c} & -\frac{11}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{11}{2} & \frac{11}{2N_c}
\end{pmatrix}
$$

with

$$
C_F = \frac{N_c^2 - 1}{2\,N_c}\,. \tag{A.2}
$$

The photonic corrections are

$$
r^e - \tilde{r}^e = \frac{1}{4} \times \tag{A.3}
$$

$$
\times \begin{pmatrix}
\frac{22}{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{22}{9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{9} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{11}{9} \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{9} & 0 & 0 \\
\frac{4N_c}{81} & \frac{28}{81} & \frac{-28-20\tilde{n}N_c}{81} & \frac{-4N_c+4\tilde{n}}{81} & \frac{18-20\tilde{n}N_c}{81} & \frac{-32\tilde{n}}{81} & \frac{9-20\overline{n}N_c}{81} & \frac{-32\overline{n}}{81} & \frac{14-20\overline{n}N_c}{81} & \frac{2N_c+4\overline{n}}{81} \\
0 & 0 & 0 & 0 & 0 & \frac{2}{9} & 0 & \frac{1}{9} & 0 & 0 \\
\frac{4N_c}{81} & \frac{28}{81} & \frac{22}{9} & \frac{-4N_c+4\tilde{n}}{81} & \frac{-20\tilde{n}N_c}{81} & \frac{-32\tilde{n}}{81} & \frac{-20\tilde{n}N_c}{81} & \frac{-32\overline{n}}{81} & \frac{113-20\overline{n}N_c}{81} & \frac{2N_c+4\overline{n}}{81} \\
0 & 0 & \frac{-28-20N_c\overline{n}}{81} & \frac{22}{9} & 0 & 0 & 0 & 0 & 0 & \frac{11}{9}
\end{pmatrix}
$$

with

$$
n = n_u + n_d\,; \qquad \tilde{n} = n_u - \frac{n_d}{2} \qquad \text{and} \qquad \overline{n} = n_u + \frac{n_d}{4} \tag{A.4}
$$

and n_u the number of up-like light-quarks and n_d the number of down-like light-quarks.

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